CONTENT STANDARD 6: Spatial Relationships and Geometry

Students will analyze and use spatial relationships and basic concepts of geometry to construct, draw, describe and compare geometric models and their transformations, and use geometric relationships and patterns to solve problems.

K-12 PERFORMANCE STANDARDS

Educational experiences in **Grades K-4** will assure that students:

- describe, model, draw and classify shapes;
- investigate and predict the results of combining, subdividing and changing shapes;
- identify and use geometric shapes in various orientations, including rotations, reflections and translations;
- use real-life experiences, concrete objects and technology to explore and understand properties of 2- and 3-dimensional geometric shapes; and
- explore relationships among and properties of shapes, such as congruence, similarity and symmetry.

Educational experiences in **Grades 5-8** will assure that students:

- investigate, explore and describe the geometry in nature and real-world applications;
- identify, visualize, model, describe and compare properties of and relationships among 2- and 3-dimensional shapes;
- describe and use fundamental concepts and properties of, and relationships among, points, lines, planes, angles and shapes, including incidence, parallelism, perpendicularity, congruence, similarity and the Pythagorean theorem;
- construct, analyze and apply the effects of reflections, translations, rotations and dilations on various shapes;
- relate 2- and 3-dimensional geometry using shadows, perspectives, projections and maps; and
- solve real-world problems using geometric concepts.

Educational experiences in **Grades 9-12** will assure that students:

- use transformations, coordinates and vectors and appropriate computer software to explore and develop an understanding of Euclidean geometry;
- deduce properties of, and relationships among, figures from given assumptions;
- develop an understanding of an axiomatic system through geometric investigations, making conjectures, formulating arguments and constructing proofs;
- understand and analyze the geometry of three-dimensional shapes and their cross-sections;
- solve real-world and mathematical problems using geometric models; and
- interpret algebraic equations and inequalities geometrically, and describe geometric objects algebraically.

ILLUSTRATIVE TASKS AND EXPERIENCES

As part of ongoing mathematics instruction in Grades K-4, students should have instructional experiences like the following:

1. Party Seating

Ask students to find the best way to seat 50 people coming to a party in the school cafeteria. The cafeteria has 14 rectangular tables that can be used. Each table seats three people on each side and one person at each end, except where the tables are arranged side to side or end to end. Ask students to draw the plans they come up with and tell which arrangement they think is best and why.

2. The Greedy Triangle

Use the book, *The Greedy Triangle*, by Marilyn Burns (Scholastic, 1995), when teaching about the names of various geometric shapes. The triangle is very unhappy with its shape and goes to the local "shifter" to get the shape changed. Throughout the story the triangle changes into many different shapes simply by adding more sides. The names of various shapes are discussed along with information as to where these shapes might be found in the world around us. As shapes get shifted, students draw the new shapes and predict their names and where they can be found.

3. Alphabet Symmetry

Ask students to create a set of block letters for the letters of the alphabet. Sort the letters into the following groups: those letters that can be folded over horizontally onto themselves, those letters that can be folded over vertically onto themselves, those letters that can be folded over both horizontally and vertically onto themselves, and those that can never be folded over onto themselves. Discuss the common features of the letters in each of the four sets.

As part of ongoing mathematics instruction in Grades 5-8, students should have instructional experiences like the following:

1. Great Geometry Scavenger Hunt

Provide students with the following list of geometric terms:

square	regular polygon
triangle	irregular polygon
circle	sphere
rectangle	cube
parallelogram	cylinder
rhombus	cone
triangle: acute, right, obtuse	pyramid

Working in groups, ask students to be sure they can explain in their own words what is meant by each of the terms and state the properties of each shape. Ask students to come to a group consensus on a good definition for each term and to write down their definitions. After a discussion, use the group definitions to arrive at a class definition for each term.

Once these definitions have been agreed upon, begin the Great Geometry Scavenger Hunt by asking each student to identify at least three specific examples of each shape in their environment – at home, in school or in the community. Compile a class list of all the real-world examples of these shapes that are found.

Extension: Identify objects that are similar, congruent and symmetrical.

2. Polynominoes

Each student is given six square tiles. Students are told that they are going to study the number of unique shapes that can be formed using up to six tiles that can only be connected exactly from side to side. Begin by constructing a two-column table headed "Number of Squares" and "Number of Unique Shapes." Begin with one square and clarify that there is only one shape – that square, no matter how rotated – that can be formed. Ask students to take two squares and determine how many unique shapes can be formed. Build agreement that there is only one such shape – the domino rectangle. Continue in this manner to build agreement on what the two trinominoes (three squares) and the five tetrominoes (four squares) look like. Now set students loose on finding all possible pentominoe shapes – there are 12 – and for extra credit, all possible hexominoe shapes – there are 35! Discuss the various processes students used to find the different shapes and how they knew they had them all.

3. Pegs And Holes

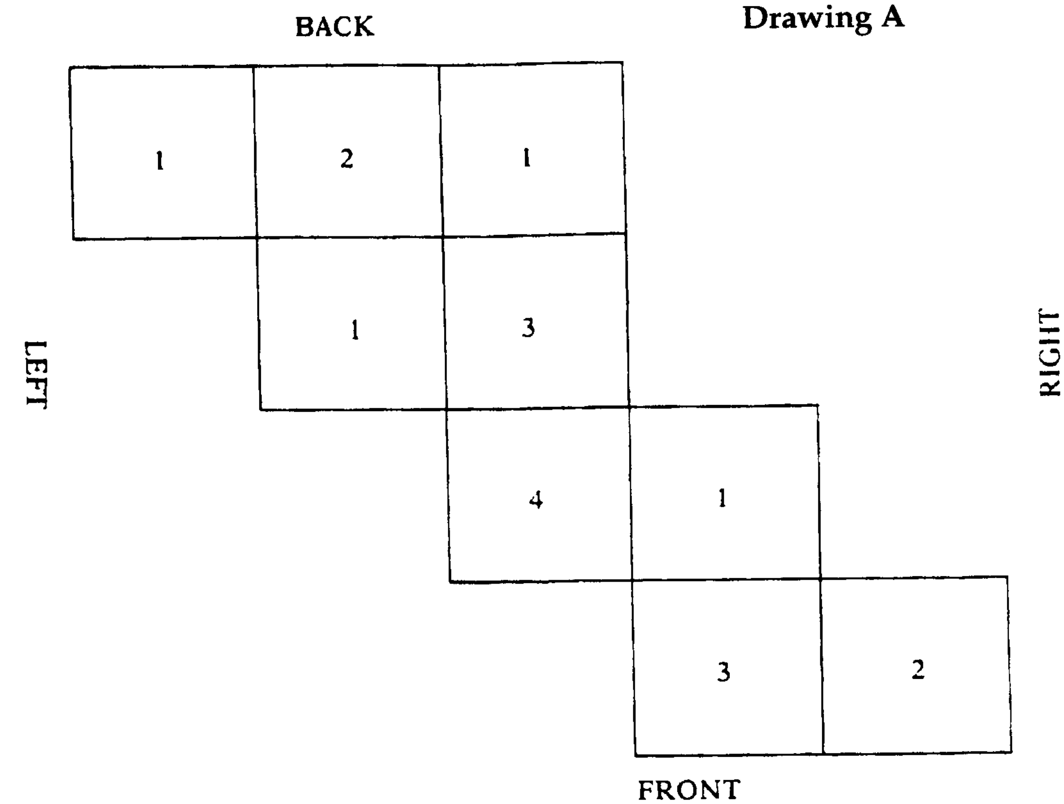
Students are asked to explain – mathematically – which is the better fit: a square peg in a round hole or a round peg in a square hole.

4. 3-D Drawings

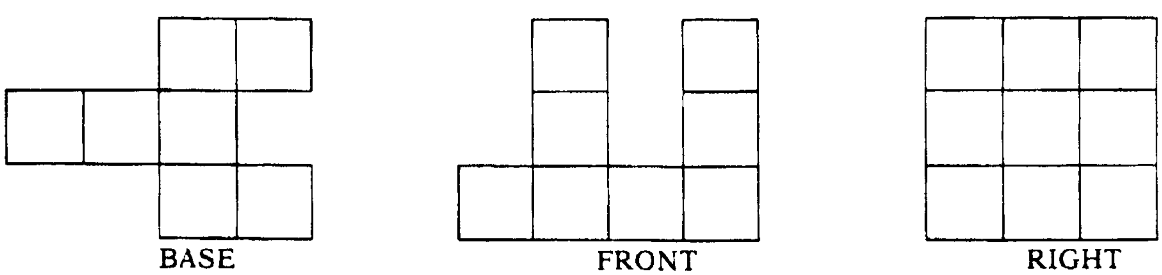
Students are given a supply of colored cubes, a 5x5 mat upon which they can construct three-dimensional "buildings" with the cubes, and grid paper upon which they can record front, side and top views of their "buildings."

- Allow students time for free exploration with their cubes, perhaps asking them to use 15 cubes and their mats to construct different one-, two- and three-story "buildings" and to share their designs with other students.
- Show students the design notation that uses numbers on the mat to represent the number of cubes stacked on each square. Present a block design like the one shown in drawing A on page 88 for students to construct individually or in pairs.
- Ask students to use grid paper to draw the front, back, right, left, top and bottom views of their block designs and to look for patterns among these views.
- Continue to ask students to move from 3-D cube construction to front, side and top views of their constructions.
- Reverse the process by giving students the three 2-dimensional views of a mystery "building" like those shown in drawing B on page 88, and ask them to create and record the construction.
- Extend this work using "Building Views" software from *Sunburst.*

Drawing A



Drawing B

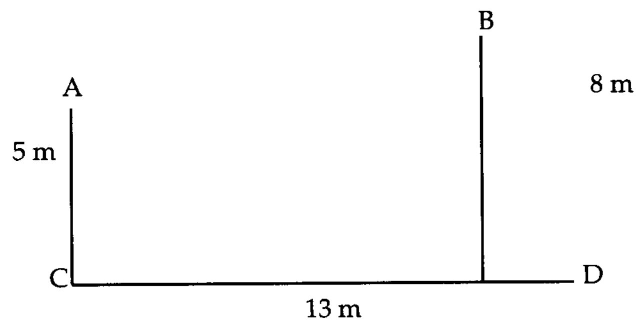


From *The Middle Grades Math Project: Spatial Visualization*, by Glenda Lappen, et al; Copyright © 1986 by Addison-Wesley Publishing Co., Inc. Reprinted by Permission.

As part of ongoing mathematics instruction in Grades 9-12, students should have instructional experiences like the following:

1. The Shortest Route

Ask students to use geometry software tools to investigate the following problem: In the figure below, a TV game show contestant must run from her initial starting position "A" to a long table "CD" that is covered with chocolate cream pies. The table is 13 meters long and is placed 5 meters from "A". After picking up a pie from the table, the contestant is to race to her partner, who is 8 meters from the table at "B", and give him a face full of pie. What is the shortest distance in which she can accomplish this feat? Where along the table should she grab a pie? How can the result be generalized to any two distances "a" and "b" from a table "c" units long?



2. The Staircase

Students who have been working with the Pythagorean Theorem are told that the building code requires that staircases have a slope of no more than 0.80 and no less than 0.55. In addition, the rise plus the run on each step must be between 17 and 18 inches. Ask students to design a staircase that rises a total of 11 feet and meets the building code. Students should complete sketches of the staircase, showing all necessary measurements, and showing that the building code is met.

3. Kites

Students are told that "kites" are quadrilaterals with two pairs of congruent adjacent sides. Ask students, working in groups, to make conjectures about four different properties of all kites, and to demonstrate that their conjectures are correct.

4. A Triangle Exploration

Give students, orally or in writing, the following instructions:

1. Draw an arbitrary triangle ABC and mark the midpoints A', B' and C' of sides BC, CA and AB respectively.
2. Choose an arbitrary point X on AB and draw a line through X parallel to C' A', meeting BC at point P.
3. Through P draw the line parallel to A' B', meeting CA at point Q.
4. Through Q draw the line parallel to B' C', meeting AB at point X'.

Having completed the construction with a straight edge and compass or on a computer, discuss the following:

- What is your guess about the relationship of lengths X' C' and XC'? Does your conjecture change if you select a different placement for X?
- Prove your conjecture about the lengths of X' C' and XC'.
- Beginning with point X' on AB, repeat the construction to obtain point X". What is the relationship of point X" to X? How do you know?
- Suppose the entire construction is repeated with A', B' and C' being the feet of altitudes of the triangle rather than midpoints of sides. What happens?
- Extend the results of this work to any three points on the sides of a triangle.

PROTOTYPE ASSESSMENTS AND SAMPLES OF STUDENT WORK

As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 4, all students should be expected to complete work like the sample below:

PROBLEM SOLVING WITH TRIANGLES ON DOT PAPER GEOBOARDS

Make a triangle with a geoband on a 5 x 5 geoboard, or with a pencil on dot paper.

- How many pegs does your geoband touch? How many dots or pegs border the triangle on the dot paper?
- Make three different triangles, one that touches exactly three pegs or dots, one that touches exactly four pegs or dots, and one that touches exactly five pegs or dots.
- Can you make a triangle that touches only two pegs? Explain why this does or does not work.
- Is it possible to make a triangle that has the same number of pegs on two sides? On three sides? Discuss this with a partner.

Extend this activity with other shapes and keep track of three things: the number of sides, the number of pegs inside, and the number of pegs on the perimeter.

PROBLEM SOLVING WITH GEOBOARDS

Make a triangle on a 5 x 5 geoboard, or on dot paper.

1 • How many pegs (or dots) does your geo-band touch? (or How many dots/pegs border the triangle on the dot paper?)

2 • Make three triangles so that the geo-band touches 3 pegs, 4 pegs, and 5 pegs.

3 • Can you make a triangle that touches only 2 pegs? Explain why this does, or does not, work. No, Because A triangle has to have 3 virteces And the least # of Pegs you could use is 3

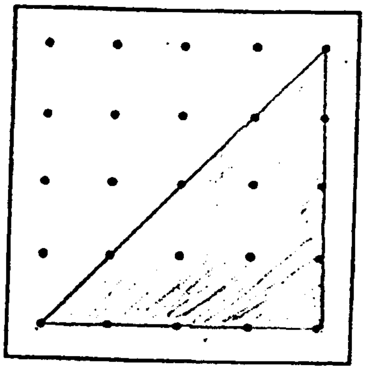
4 • Is it possible to make a triangle that has the same number of pegs on 2 sides? on 3 sides? Discuss this with your partner. Yes, Yes

5 Extend this with other shapes remembering three things: the number of sides, the number of pegs inside, and the number of pegs on the boundary.

# of sides	# of vectices	Name	Drawing
3	3	Triangle	
4	4	square	
5	5	pentagon	
6	6	hexagon	
7	7	septagon	

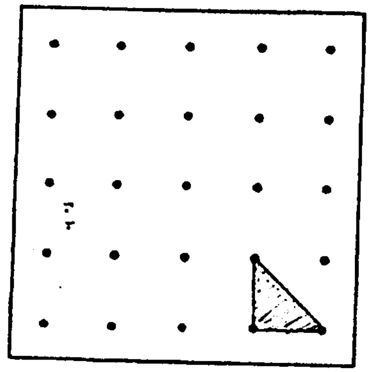
Geoboard Grids — Master

1

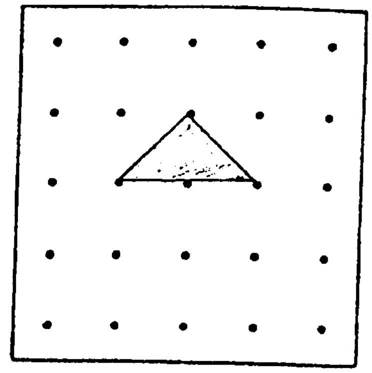


12 Pegs

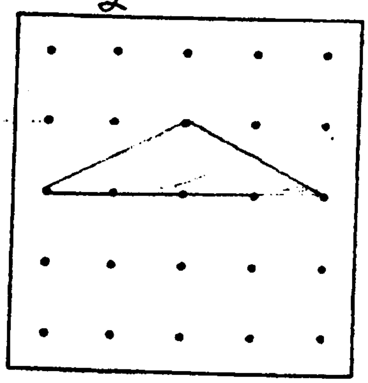
2



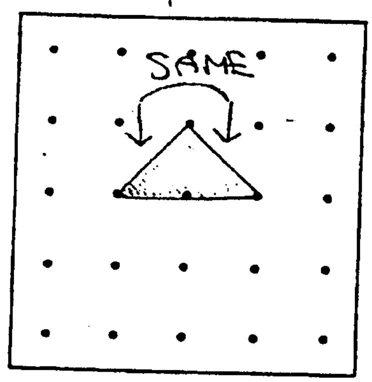
2



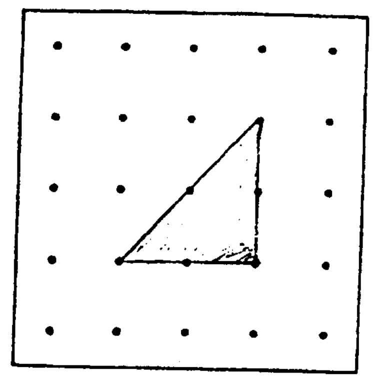
2



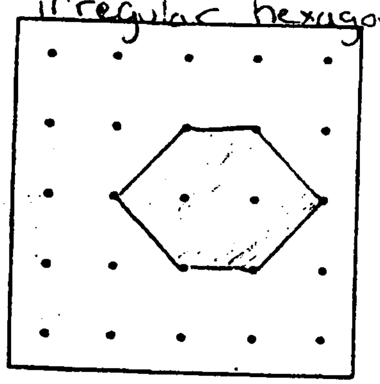
4



4

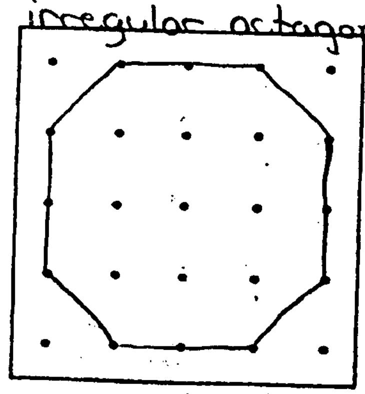


irregular hexagon



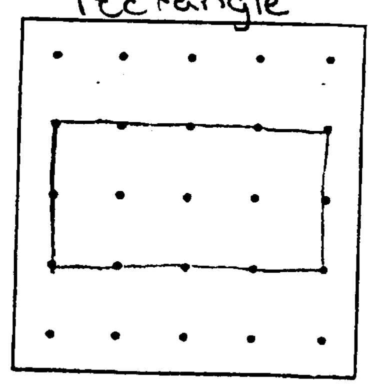
6 Sides 2inside
6 borders

irregular octagon



8 sides
9 inside
12 borders

rectangle



4 sides
3 inside
12 borders

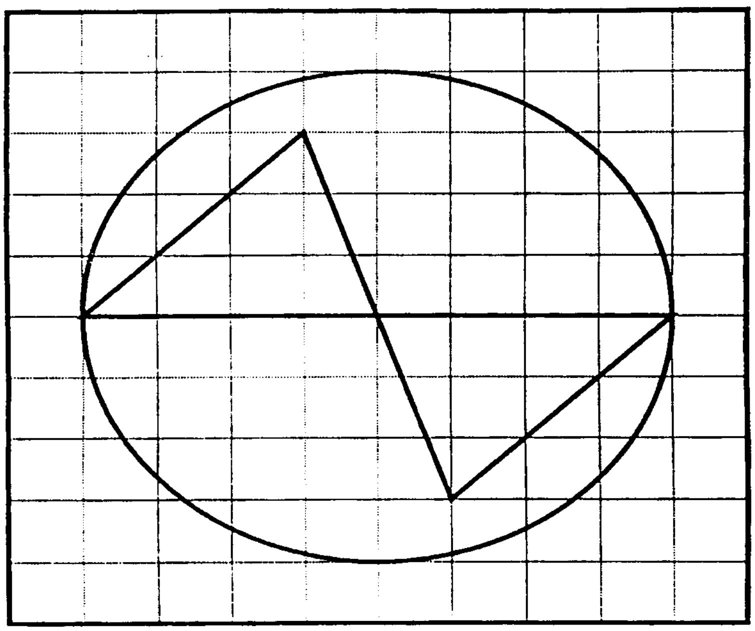
As a result of an instructional program in mathematics like that described in this guide, by the end of Grade 8, all students should be expected to complete work like the sample below:

T-SHIRT DESIGN

The aims of this assessment are to provide the opportunity for you to:
• communicate mathematically about geometric patterns
• locate shapes on the plane

The design below, including the 10 by 10 grid, is going to be used on a math team T-shirt. You accidentally took the original design home, and your friend, Chris, needs it tonight. Chris has no fax machine, but has a 10 by 10 grid just like yours (see Chris' grid on the next page). You must call Chris on the telephone and tell him very precisely how to draw the design on a his grid.

Prepare for the phone call by writing out your directions clearly, ready to read over the telephone.



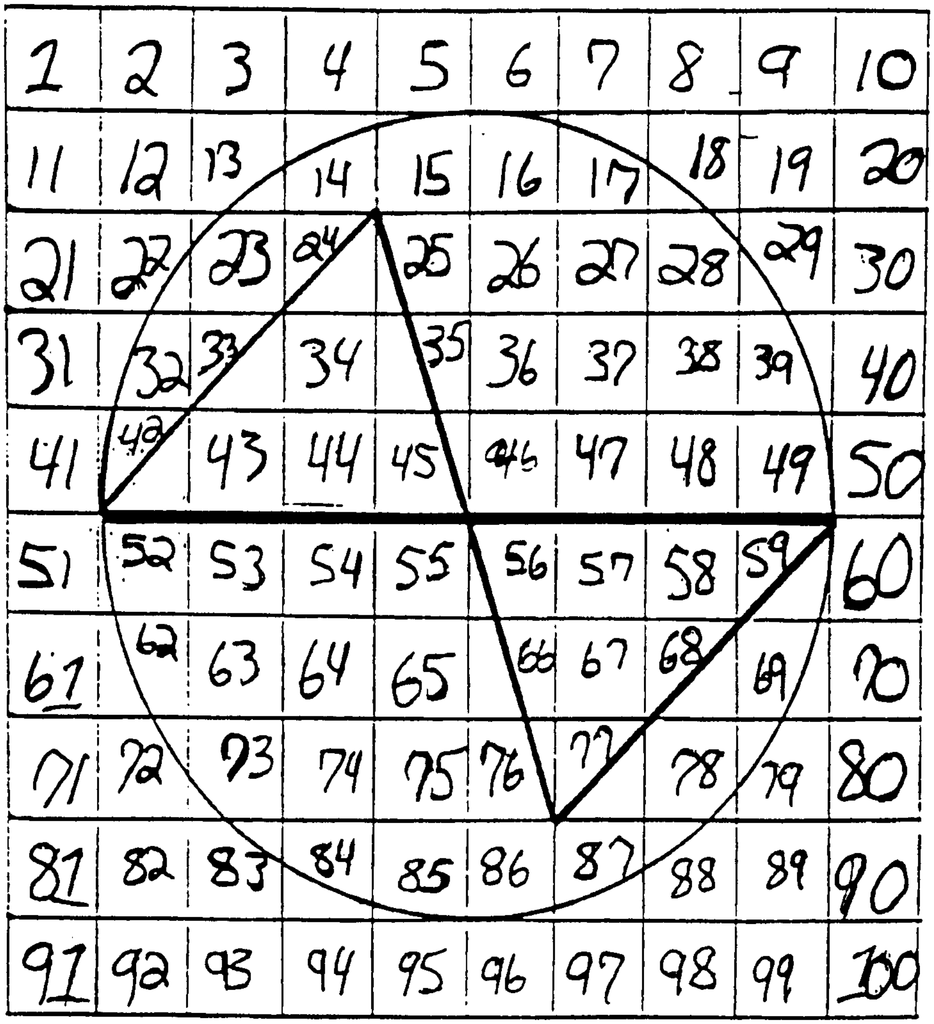
Write your directions on another sheet of paper.

From Balanced Assessment.
White Plains, N.Y., 1995.
Used with permission.

T-Shirt Design

The design below, including the 10 by 10 grid, is going to be used on a math team t-shirt. You accidentally took the original design home, and your friend needs it tonight. Your friend has no fax machine, so you will have to call her on the telephone and tell her <u>very precisely</u> how to draw the design on a piece of graph paper.

Prepare for the phone call by writing out your directions clearly, ready to read over the telephone.



Write your directions on another sheet of paper.

"T-Shirt Design"

1. Make a 10 by 10 grid with the
boxes ½ an inch wide and ½ an inch tall.
2. Number them 1 to 100 starting from
the top left-hand corner.
3. At the line seperating 42 from 52
make a 4-inch line going from
boxes 42 and 52 across to 49 and 59
4. Make a Qaurter of a circle starting
at the left edge of the mid line going to
the center of 5,6, 15, and 16.
5. Make another Qaurter of a circle
starting at the center of 5, 6, 15, and 16
and ending at the center of 49, 50, 59, and
60.
6. Make another Qaurter of a Circle
starting at the center of 49, 50, 59, and
60 and ending at the center of 85, 86, 95 and
7. Make another Qaurter of a circle start
at the center of 85, 86, 95 and 96 and ending at
the center of 41, 42, 51, and 52
which is where you started. You should
now have a circle that is 4-inces
wide and 4-inces tall. If you
have a line from the too of the circle